

On an Irreducible Theory of Complex Systems

Victor Korotkikh* and Galina Korotkikh†

*Faculty of Business and Informatics
Central Queensland University
Mackay, Queensland 4740, Australia*

In the paper we present results to develop an irreducible theory of complex systems in terms of self-organization processes of prime integer relations. Based on the integers and controlled by arithmetic only the self-organization processes can describe complex systems by information not requiring further explanations. Important properties of the description are revealed. It points to a special type of correlations that do not depend on the distances between parts, local times and physical signals and thus proposes a perspective on quantum entanglement. Through a concept of structural complexity the description also computationally suggests the possibility of a general optimality condition of complex systems. The computational experiments indicate that the performance of a complex system may behave as a concave function of the structural complexity. A connection between the optimality condition and the majorization principle in quantum algorithms is identified. A global symmetry of complex systems belonging to the system as a whole, but not necessarily applying to its embedded parts is presented. As arithmetic fully determines the breaking of the global symmetry, there is no further need to explain why the resulting gauge forces exist the way they do and not even slightly different.

PACS numbers: 89.75.-k, 89.75.Fb

I. INTRODUCTION

Complex systems profoundly change human activities of the day and may be of strategic interest. As a result, it becomes increasingly important to have confidence in the theory of complex systems. Ultimately, this calls for clear explanations why the foundations of the theory are valid in the first place. The ideal situation would be to have an irreducible theory of complex systems not requiring a deeper explanatory base in principle. But the question arises: where could such a theory come from, when even the concept of spacetime is questioned [1] as a fundamental entity. As a possible answer it is suggested that the concept of integers may take responsibility in the search for an irreducible theory of complex systems [2]. It is shown that complex systems can be described in terms of self-organization processes of prime integer relations [2], [3]. Based on the integers and controlled by arithmetic only the self-organization processes can describe complex systems by information not requiring further explanations. This gives the possibility to develop an irreducible theory of complex systems. In the paper we present results to progress in this direction.

II. INVARIANT QUANTITIES OF A COMPLEX SYSTEM AND UNDERLYING CORRELATIONS

To understand a complex system we consider the dynamics of the elementary parts and focus on the correla-

tions preserving certain quantities of the complex system [2], [3].

Let I be an integer alphabet and

$$I_N = \{x = x_1 \dots x_N, x_i \in I, i = 1, \dots, N\}$$

be the set of sequences of length $N \geq 2$. We consider N elementary parts $P_i, i = 1, \dots, N$ with the state of an element P_i specified in its reference frame by a generalized coordinate $x_i \in I, i = 1, \dots, N$ (for example, the position of the element P_i in space) and the state of the elements by a sequence $x = x_1 \dots x_N \in I_N$.

For a geometric representation of the sequences we use piecewise constant functions. Let $\varepsilon > 0$ and $\delta > 0$ be length scales of a two-dimensional lattice. Let

$$\rho_{m\varepsilon\delta} : x \rightarrow f$$

be a mapping that realizes the geometric representation of a sequence $x = x_1 \dots x_N \in I_N$ by associating it with a function $f \in W_{\varepsilon\delta}[t_m, t_{m+N}]$, denoted $f = \rho_{m\varepsilon\delta}(x)$, such that

$$f(t_m) = x_1\delta, f(t) = x_i\delta, t \in (t_{m+i-1}, t_{m+i}], i = 1, \dots, N,$$

$$t_i = i\varepsilon, i = m, \dots, m + N$$

and whose integrals $f^{[k]}$ satisfy the condition

$$f^{[k]}(t_m) = 0, k = 1, 2, \dots,$$

where m is an integer. The sequence $x = x_1 \dots x_N$ is called a code of the function f and denoted $c(f)$.

By using the geometric representation for a state $x = x_1 \dots x_N \in I_N$ of the elements $P_i, i = 1, \dots, N$ we define quantities of the complex system as the integrals

$$f^{[k]}(t_{m+N}) = \int_{t_m}^{t_{m+N}} f^{[k-1]}(t) dt, f^{[0]} = f, k = 1, 2, \dots$$

*Electronic address: v.korotkikh@cqu.edu.au

†Electronic address: g.korotkikh@cqu.edu.au

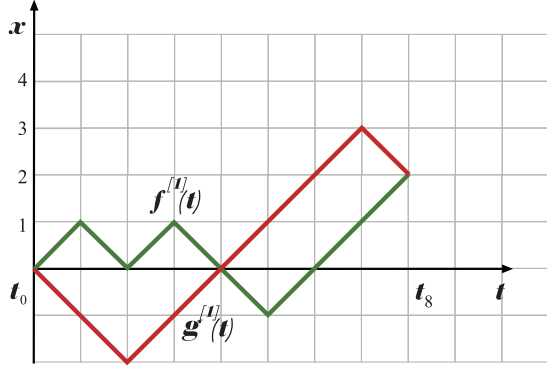


FIG. 1: For states $x = +1 - 1 + 1 - 1 - 1 + 1 + 1 + 1$ and $x' = -1 - 1 + 1 + 1 + 1 + 1 + 1 - 1$ the first integrals are equal $f^{[1]}(t_8) = g^{[1]}(t_8)$, where $f = \rho_{m\varepsilon\delta}(x)$, $g = \rho_{m\varepsilon\delta}(x')$ and $m = 0$, $\varepsilon = 1$, $\delta = 1$. It turns out that the second integrals are also equal $f^{[2]}(t_8) = g^{[2]}(t_8)$, but the third integrals are not $f^{[3]}(t_8) \neq g^{[3]}(t_8)$. Thus, two quantities remain invariant and $C(x, x') = 2$.

of a function $f^{[0]} = f = \rho_{m\varepsilon\delta}(x) \in W_{\varepsilon\delta}[t_m, t_{m+N}]$ [2].

Remarkably, the integer code series [4] expresses the definite integral

$$f^{[k]}(t_{m+N}) = \sum_{i=0}^{k-1} \alpha_{kmi} ((m+N)^i x_1 + \dots + (m+1)^i x_N) \varepsilon^k \delta$$

of a function $f \in W_{\varepsilon\delta}([t_m, t_{m+N}])$ by using the code $c(f) = x_1 \dots x_N$ of the function f , powers

$$(m+N)^i, \dots, (m+1)^i, \quad i = 0, \dots, k-1$$

of integers $(m+N), \dots, (m+1)$ and combinatorial coefficients

$$\alpha_{kmi} = ((-1)^{k-i-1} (m+1)^{k-i} + (-1)^{k-i} m^{k-i}) / (k-i)! i!,$$

where $k \geq 1$ and $i = 0, \dots, k-1$.

We consider the correlations conserving the quantities

$$f^{[k]}(t_{m+N}) = g^{[k]}(t_{m+N}), \quad k = 1, \dots, C(x, x'), \quad (1)$$

$$f^{[C(x, x') + 1]}(t_{m+N}) \neq g^{[C(x, x') + 1]}(t_{m+N}), \quad (2)$$

as elements $P_i, i = 1, \dots, N$ change from one state $x = x_1 \dots x_N \in I_N$ to another $x' = x'_1 \dots x'_N \in I_N$, where $f = \rho_{m\varepsilon\delta}(x)$, $g = \rho_{m\varepsilon\delta}(x')$ [2]. The conditions (1) and (2) suggest that as a complex system changes, its parts are correlated to preserve $C(x, x')$ of the quantities (Figure 1).

It is worthwhile to mention that the conditions (1) and (2) specify a symmetry transformation possibly determining the equation of motion of the complex system. Therefore, once the number $C(x, x')$ of invariants is viewed as a variable to describe an order, we may have a scheme classifying equations of motion.

It is proved [2] that $C(x, x') \geq 1$ of the quantities of a complex system remain invariant, if and only if $C(x, x')$ equations take place

$$(m+N)^{C(x, x')-1} \Delta x_1 + \dots + (m+1)^{C(x, x')-1} \Delta x_N = 0$$

$$(m+N)^1 \Delta x_1 + \dots + (m+1)^1 \Delta x_N = 0$$

$$(m+N)^0 \Delta x_1 + \dots + (m+1)^0 \Delta x_N = 0 \quad (3)$$

characterizing in view of an inequality

$$(m+N)^{C(x, x')} \Delta x_1 + \dots + (m+1)^{C(x, x')} \Delta x_N \neq 0,$$

the correlations between the parts of the complex system, where $\Delta x_i = x'_i - x_i$ are the changes of the elements $P_i, i = 1, \dots, N$ in their reference frames and m is an integer.

The coefficients of the system of linear equations become the entries of the Vandermonde matrix, when the number of the equations is N . This fact is important in order to prove that $C(x, x') < N$ [2].

The equations (3) present a special type of correlations that do not have reference to the distances between parts, local times and physical signals. The space and non-signaling aspects of the correlations are familiar from explanations of quantum correlations through entanglement [5]. The time aspect of the correlations may suggest something new into the agenda.

III. SELF-ORGANIZATION PROCESSES OF PRIME INTEGER RELATIONS AND THEIR GEOMETRIZATION

The equations (3) can be also viewed as identities. Their analysis reveals hierarchical structures of prime integer relations in the description of complex systems [2], [3] (Figure 2). In the context of the hierarchical structures it may be useful to investigate whether the Ward identities and their generalizations [6] could be presented in a more explicit form.

The hierarchical structures underlying equations (3) may be in a certain superposition with each other. Namely, a prime integer relation may simultaneously belong to a number of the hierarchical structures. The measurements specifying the behavior of the part controlled by the prime integer relation can propagate through the superposition to associated prime integer relations and thus effect other parts of the complex system.

Through the hierarchical structures a new type of processes, i.e., the self-organization processes of prime integer relations, is revealed [2]. Starting with integers as the elementary building blocks and following a single

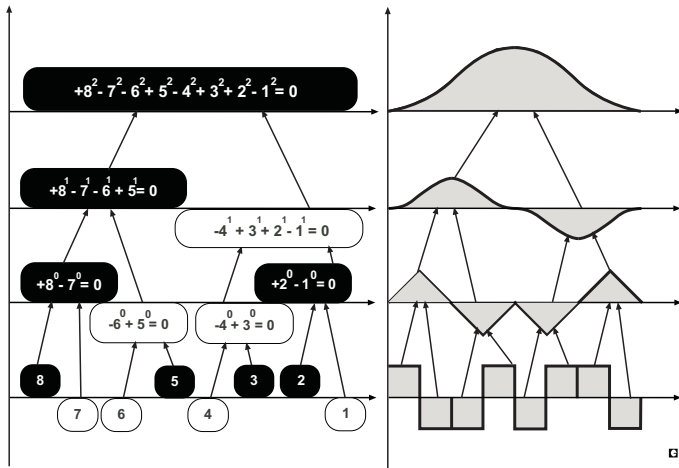


FIG. 2: The left side shows one of the hierarchical structures of prime integer relations, when a complex system has $N = 8$ elements $P_i, i = 1, \dots, 8$, $x = 00000000$, $x' = +1 - 1 - 1 + 1 - 1 + 1 + 1 - 1$, $m = 0$ and $C(x, x') = 3$. The hierarchical structure is built by a self-organization process of prime integer relations and controls a correlation structure of the complex system. The right side presents an isomorphic hierarchical structure of geometric patterns. On scale level 0 eight rectangles specify the dynamics of the elements $P_i, i = 1, \dots, 8$. The boundary curves of the geometric patterns describe the dynamics of the parts. All geometric patterns are symmetric and their symmetries are interconnected. The symmetry of a geometric pattern is global and belongs to a corresponding part as a whole.

principle, a self-organization process makes up the prime integer relations of a level of a hierarchical structure from the prime integer relations of the lower level (Figure 2).

A prime integer relation is made as an inseparable object: if even one of the prime integer relations is not included, then the rest of the prime integer relations can not form an integer relation. In other words, each and every prime integer relation involved in the formation of a prime integer relation is critical.

The prime integer relations set limits of causality in the sense that complex systems controlled by separate prime integer relations have no effect on each other. Namely, if a part of a complex system changes, then, in accordance with the prime integer relation, the other parts change no matter how far they may be from each other. At the same time the prime integer relation has no causal power to influence parts of complex systems controlled by separate prime integer relations.

An analogy between the prime integer relation and the light cone may be suggested. While in spacetime events can be only observed in light cones, in our approach events can only happen through the hierarchical structure of the prime integer relations.

By using the integer code series [4] the prime integer relations can be geometrized as two-dimensional geometric patterns and the self-organization processes can be isomorphically expressed by transformations of the patterns

[2]. As it becomes possible to measure a prime integer relation by an isomorphic geometric pattern, quantities of the prime integer relation and a complex system it describes can be defined by quantities of the geometric pattern such as the area and the length of its boundary curve (Figure 2).

Due to the isomorphism, the structure and the dynamics of a complex system are combined. As self-organization processes of prime integer relations determine the correlation structure of a complex system, transformations of corresponding geometric patterns may characterize its dynamics in a strong scale covariant form [2], [3].

IV. STRUCTURAL COMPLEXITY IN OPTIMALITY CONDITION OF COMPLEX SYSTEMS AND OPTIMAL QUANTUM ALGORITHMS

Despite different origin complex systems have much in common and are investigated to satisfy universal laws. Our description points out that the universal laws may originate not from forces in spacetime, but through arithmetic.

There are many notions of complexity introduced in the search to communicate the universal laws into theory and practice. The concept of structural complexity is defined to measure the complexity of a system in terms of self-organization processes of prime integer relations [2]. In particular, as self-organization processes of prime integer relations progress from a level to the higher level, the system becomes more complex, because its parts at the level are combined to make up more complex parts at the higher level. Therefore, the higher the level self-organization processes progress to, the greater is the structural complexity of a corresponding complex system.

Existing concepts of complexity do not explain in general how the performance of a complex system may depend on its complexity. To address the situation we conducted computational experiments to investigate whether the concept of structural complexity could make a difference [7], [8].

A special optimization algorithm, as a complex system, was developed to minimize the average distance in the travelling salesman problem. Remarkably, for each problem the performance of the algorithm was concave. As a result, the algorithm and a problem were characterized by a single performance optimum. The analysis of the performance optimums for all problems tested revealed a relationship between the structural complexity of the algorithm and the structural complexity of the problem approximating it well enough by a linear function [8].

The results of the computational experiments suggest the possibility of a general optimality condition of complex systems:

A complex system demonstrates the optimal perfor-

mance for a problem, when the structural complexity of the system is in a certain relationship with the structural complexity of the problem.

The optimality condition presents the structural complexity of a system as a key to its optimization. Indeed, according to the optimality condition the optimal result can be obtained as long as the structural complexity of the system is properly related with the structural complexity of the problem. From this perspective the optimization of a system should be primarily concerned with the control of the structural complexity of the system to match the structural complexity of the environment.

The computational results also indicate that the performance of a complex system may behave as a concave function of the structural complexity. Once the structural complexity would be controlled as a single entity, the optimization of a complex system could be potentially reduced to a one-dimensional concave optimization irrespective of the number of variables involved its description.

In the search to identify a mathematical structure underlying optimal quantum algorithms the majorization principle emerges as a necessary condition for efficiency in quantum computational processes [9]. We find a connection between the optimality condition and the majorization principle in quantum algorithms.

According to the majorization principle in an optimal quantum algorithm the probability distribution associated to the quantum state has to be step-by-step majorized until it is maximally ordered. This means that an optimal quantum algorithm works in such a way that the probability distribution p_{k+1} at step $k+1$ majorizes p_k at step k . There are special conditions in place for the probability distribution p_{k+1} to majorize the probability distribution p_k with intuitive meaning that the distribution p_k is more disordered than p_{k+1} [9].

In our description the algorithm revealing the optimality condition also uses a similar principle, but based on the structural complexity. The algorithm tries to work in such a way that the structural complexity \mathbf{C}_{k+1} of the algorithm at step $k+1$ majorizes \mathbf{C}_k at step k . The concavity of the algorithm's performance suggests efficient means to find optimal solutions [8].

V. GLOBAL SYMMETRY OF COMPLEX SYSTEMS AND GAUGE FORCES

Our description presents a global symmetry of complex systems through the geometric patterns of prime integer relations and their transformations. It belongs to the system as a whole, but does not necessarily apply to its embedded parts. The differences between the behaviors of the parts may be interpreted through the existence of gauge forces acting in their reference frames. As arithmetic fully determines the breaking of the global

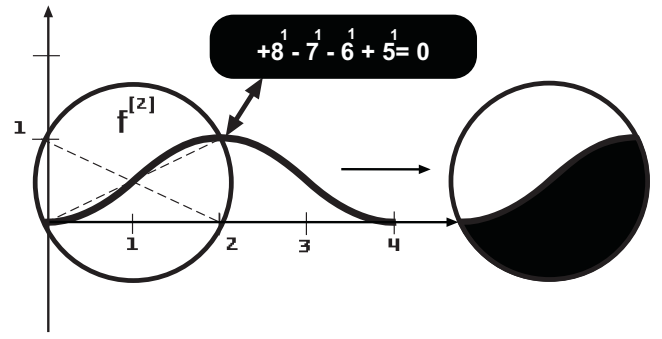


FIG. 3: The geometric pattern of the part $(P_1 \leftrightarrow P_2) \leftrightarrow (P_3 \leftrightarrow P_4)$. From above the pattern is limited by the boundary curve, i.e., the graph of the second integral $f^{[2]}(t)$, $t_0 \leq t \leq t_4$ of the function f defined on scale level 0 (Figure 2), where $t_i = i\varepsilon$, $i = 1, \dots, 4$, $\varepsilon = 1$, and it is restricted by the t axis from below. The geometric pattern is isomorphic to the prime integer relation $+8^1 - 7^1 - 6^1 + 5^1 = 0$ and determines the dynamics. If the part deviates from this dynamics even slightly, then some of the correlation links provided by the prime integer relation disappear and the part decays. The boundary curve has a special property ensuring that the area of the geometric pattern is given as the area of a triangle: $S = \frac{HL}{2}$, where H and L are the height and the length of the geometric pattern. In the figure $H = 1$ and $L = 4$, thus $S = 2$. The property is illustrated in yin-yang motifs.

symmetry, there is no further need to explain why the resulting gauge forces exist the way they do and not even slightly different.

Let us illustrate the results by a special self-organization process of prime integer relations [2], [3]. The left side of Figure 2 shows a hierarchical structure of prime integer relations built by the process. It controls a correlation structure of a complex system with states of $N = 8$ elements P_i , $i = 1, \dots, 8$ given by sequences

$$x = 00000000, \quad x' = +1 - 1 - 1 + 1 - 1 + 1 + 1 - 1$$

and $m = 0$. The sequence x' is the initial segment of length 8 of the Prouhet-Thue-Morse (PTM) sequence starting with $+1$. There is an ensemble of self-organization processes and thus correlation structures forming the correlation structure of the complex system associated with the states x and x' . The self-organization process we consider is only one of them.

The right side of Figure 2 presents an isomorphic hierarchical structure of geometric patterns. The boundary curves of the geometric patterns determine the dynamics of parts of the complex system. Quantities of a geometric pattern, such as its area and the length of the boundary curve, define quantities of a corresponding part of the complex system. The quantities of the parts are interconnected through the transformations of the geometric patterns.

The area of the geometric pattern has a remarkable property. Although a geometric pattern on scale level $\mathcal{N} > 1$ is bounded by an intricate curve, its area S , nev-

ertheless, can be expressed as the area of a triangle by

$$S = \frac{HL}{2},$$

where H and L are the height and the length of the geometric pattern (Figures 2 and 3). The area has a strong scale covariant description: the law of geometric pattern area and corresponding quantity E is the same and in the simplest possible form for all scale levels [3].

Importantly, the quantity of a complex system specified by the height H of the geometric pattern and denoted by M is determined by the area quantities E_l and E_r of two parts, i.e., left and right, the complex system is made of (Figure 2). Namely, the height H of the geometric pattern equals one half of the sum of the areas of the geometric patterns characterizing the parts forming the complex system from the lower scale level.

Thus, we have

$$M = \frac{E_l + E_r}{2}$$

suggesting that, as the parts on scale level $\mathcal{N} - 1$ form the complex system on scale level \mathcal{N} , their E quantities are converted into the M quantity of the complex system. The M quantity is proportional and as such can be seen equivalent to the E quantity of the complex system

$$E = \frac{ML}{2}.$$

To investigate how the length ϑ of the boundary curve of a geometric pattern could be expressed in terms of its length L we try for $\varepsilon = 1$ and $\delta = 1$ a formula

$$\vartheta = \frac{\mathcal{F}L}{4}, \quad (4)$$

where

$$\mathcal{F} = 4.6692\dots$$

is the Feigenbaum constant. It turns out that in the case of the self-organization process the formula (4) works better and better as we proceed from scale level 1 to scale level 3. This can be demonstrated by comparing the length calculated numerically [3] and the same length but given by the formula.

For the geometric pattern of the part $P_1 \leftrightarrow P_2$ at scale level 1 the value of ϑ when obtained computationally is 2.8284... . The formula (4) for $\mathcal{F} = 4.6692$ and $L = 2$ gives

$$\vartheta'_1 = \frac{4.6692 \times 2}{4} = 2.3346.$$

We can calculate the error as

$$e_1 = \frac{|\vartheta_1 - \vartheta'_1|}{\vartheta_1}$$

$$= \frac{|2.8284 - 2.3346|}{2.8284} \approx 0.2.$$

For the geometric pattern of the part

$$(P_1 \leftrightarrow P_2) \leftrightarrow (P_3 \leftrightarrow P_4)$$

at scale level 2 we get

$$\vartheta_2 = 4.5911\dots$$

and by the formula (4) for $L = 4$

$$\vartheta'_2 = \frac{4.6692 \times 4}{4} = 4.6692,$$

so the error

$$e_2 = \frac{|\vartheta_2 - \vartheta'_2|}{\vartheta_2} = \frac{|4.5911 - 4.6692|}{4.5911} \approx 0.02.$$

Similarly, for the geometric pattern of the part

$$((P_1 \leftrightarrow P_2) \leftrightarrow (P_3 \leftrightarrow P_4)) \leftrightarrow ((P_5 \leftrightarrow P_6) \leftrightarrow (P_7 \leftrightarrow P_8))$$

at scale level 3 we have

$$\vartheta_3 = 9.3248\dots,$$

$$\vartheta'_3 = \frac{4.6692 \times 8}{4} = 9.3384$$

and

$$e_3 = \frac{|\vartheta_3 - \vartheta'_3|}{\vartheta_3} = \frac{|9.3248 - 9.3384|}{9.3248} \approx 0.0015.$$

Therefore, the formula (4) gives roughly a ten times better approximation to the length of the boundary curve as a transition is made from a scale level \mathcal{N} to the scale level $\mathcal{N} + 1$, where $\mathcal{N} = 1, 2$ (Figure 2).

Starting with the elements at scale level 0, the parts of the correlation structure are built scale level by scale level and thus a part of the complex system becomes a complex system itself. From Figure 2 the symmetries of the geometric patterns and their interconnection generated through integrations of the function can be seen.

We consider whether the description of the dynamics of parts of a scale level is invariant as through the formation they become embedded in a part of the higher scale level. At scale level 2 the second integral

$$f^{[2]}(t), \quad t_0 \leq t \leq t_4, \quad t_i = i\varepsilon, \quad i = 1, \dots, 4, \quad \varepsilon = 1$$

characterizes the dynamics of the part

$$(P_1 \leftrightarrow P_2) \leftrightarrow (P_3 \leftrightarrow P_4).$$

This composite part is made up of elements P_1, P_2, P_3, P_4 and parts $P_1 \leftrightarrow P_2, P_3 \leftrightarrow P_4$ embedded by the formations within its correlation structure (Figures 2 and 3).

The description of the dynamics of elements P_1, P_2, P_3, P_4 and parts $P_1 \leftrightarrow P_2, P_3 \leftrightarrow P_4$ within the part

$$(P_1 \leftrightarrow P_2) \leftrightarrow (P_3 \leftrightarrow P_4)$$

is invariant relative to their reference frames. In particular, the dynamics of elements P_1 and P_2 in a reference frame of the element P_1 is specified by

$$f^{[2]}(t) = f_{P_1}^{[2]}(t_{P_1}) = \frac{t_{P_1}^2}{2!},$$

$$t_0 = t_{0,P_1} \leq t_{P_1} \leq t_{1,P_1} = t_1, \quad (5)$$

$$f^{[2]}(t) = f_{P_1}^{[2]}(t_{P_1}) = -\frac{t_{P_1}^2}{2!} + 2t_{P_1} - 1,$$

$$t_1 = t_{1,P_1} \leq t_{P_1} \leq t_{2,P_1} = t_2.$$

The transition from the coordinate system of the element P_1 to a coordinate system of the element P_2 given by the transformation

$$t_{P_2} = -t_{P_1} - 2, \quad f_{P_2}^{[2]} = -f_{P_1}^{[2]} - 1$$

shows that the characterization

$$f_{P_2}^{[2]}(t_{P_2}) = \frac{t_{P_2}^2}{2!}, \quad t_{0,P_2} \leq t_{P_2} \leq t_{1,P_2} \quad (6)$$

of the dynamics of the element P_2 is invariant, if we compare (5) and (6).

The description is also invariant, when we consider the dynamics of the elements P_3 and P_4 in their reference frames. The dynamics of the element P_3 in the coordinate system of the element P_1 is specified by

$$f_{P_1}^{[2]}(t_{P_1}) = -\frac{t_{P_1}^2}{2!} + 2t_{P_1} - 1,$$

$$t_{2,P_1} \leq t_{P_1} \leq t_{3,P_1}.$$

The description of the dynamics of the element P_3 takes the same form

$$f_{P_3}^{[2]}(t_{P_3}) = \frac{t_{P_3}^2}{2!}, \quad t_{0,P_3} \leq t_{P_3} \leq t_{1,P_3} \quad (7)$$

as (5) and (6), when using the transformation

$$t_{P_3} = t_{P_1} - 2, \quad f_{P_3}^{[2]} = -f_{P_1}^{[2]} + 1,$$

a transition to a coordinate system of the element P_3 is made.

Similarly, the dynamics of the element P_4 in the coordinate system of the element P_1 is specified by

$$f_{P_1}^{[2]}(t_{P_1}) = \frac{t_{P_1}^2}{2!} - 4t_{P_1} + 8,$$

$$t_{3,P_1} \leq t_{P_1} \leq t_{4,P_1}.$$

The transformation

$$t_{P_4} = -t_{P_1} + 4, \quad f_{P_4}^{[2]} = f_{P_1}^{[2]}$$

leads to a coordinate system of the element P_4 to demonstrate that the description of the dynamics of the element P_4 has the same form

$$f_{P_4}^{[2]}(t_{P_4}) = \frac{t_{P_4}^2}{2!}, \quad t_{0,P_4} \leq t_{P_4} \leq t_{1,P_4},$$

as (5), (6) and (7).

Furthermore, descriptions of the dynamics of the parts $P_1 \leftrightarrow P_2$ and $P_3 \leftrightarrow P_4$ are the same relative to their coordinate systems. Namely, for the dynamics of the part $P_1 \leftrightarrow P_2$ in its reference frame we have

$$f_{P_1 \leftrightarrow P_2}^{[2]}(t_{P_1 \leftrightarrow P_2}) = \begin{cases} \frac{t_{P_1 \leftrightarrow P_2}^2}{2!}, & t_{0,P_1 \leftrightarrow P_2} \leq t_{P_1 \leftrightarrow P_2} \leq t_{1,P_1 \leftrightarrow P_2} \\ -\frac{t_{P_1 \leftrightarrow P_2}^2}{2!} + 2t_{P_1 \leftrightarrow P_2} - 1, & t_{1,P_1 \leftrightarrow P_2} \leq t_{P_1 \leftrightarrow P_2} \leq t_{2,P_1 \leftrightarrow P_2} \end{cases} \quad (8)$$

For the dynamics of the part $P_3 \leftrightarrow P_4$ in the reference frame of the part $(P_1 \leftrightarrow P_2)$ we have

$$f_{P_1 \leftrightarrow P_2}^{[2]}(t_{P_1 \leftrightarrow P_2}) = \begin{cases} -\frac{t_{P_1 \leftrightarrow P_2}^2}{2!} + 2t_{P_1 \leftrightarrow P_2} - 1, & t_{2,P_1 \leftrightarrow P_2} \leq t_{P_1 \leftrightarrow P_2} \leq t_{3,P_1 \leftrightarrow P_2} \\ \frac{t_{P_1 \leftrightarrow P_2}^2}{2!} - 4t_{P_1 \leftrightarrow P_2} + 8, & t_{3,P_1 \leftrightarrow P_2} \leq t_{P_1 \leftrightarrow P_2} \leq t_{4,P_1 \leftrightarrow P_2} \end{cases} \quad (9)$$

The description (9) takes the same form

$$f_{P_3 \leftrightarrow P_4}^{[2]}(t_{P_3 \leftrightarrow P_4}) = \begin{cases} \frac{t_{P_3 \leftrightarrow P_4}^2}{2!}, & t_{0,P_3 \leftrightarrow P_4} \leq t_{P_3 \leftrightarrow P_4} \leq t_{1,P_3 \leftrightarrow P_4} \\ -\frac{t_{P_3 \leftrightarrow P_4}^2}{2!} + 2t_{P_3 \leftrightarrow P_4} - 1, & t_{1,P_3 \leftrightarrow P_4} \leq t_{P_3 \leftrightarrow P_4} \leq t_{2,P_3 \leftrightarrow P_4} \end{cases}$$

as (8), if under the transformation

$$t_{P_3 \leftrightarrow P_4} = t_{P_1 \leftrightarrow P_2} + 2, \quad f_{P_3 \leftrightarrow P_4}^{[2]} = -f_{P_1 \leftrightarrow P_2}^{[2]} + 1,$$

we make a transition from the reference frame of the part $P_1 \leftrightarrow P_2$ to a reference frame of the part $P_3 \leftrightarrow P_4$.

Thus, as the perspective is changed from the reference frame of the part $P_1 \leftrightarrow P_2$ to the reference frame of the

part $P_3 \leftrightarrow P_4$ the description of the dynamics remains invariant.

However, at scale level 3 the description of the dynamics is not invariant. In particular, the dynamics of elements P_1 and P_2 within the part

$$((P_1 \leftrightarrow P_2) \leftrightarrow (P_3 \leftrightarrow P_4)) \leftrightarrow ((P_5 \leftrightarrow P_6) \leftrightarrow (P_7 \leftrightarrow P_8))$$

relative to a coordinate system of the element P_1 can be specified accordingly by (Figure 2)

$$f_{P_1}^{[3]}(t_{P_1}) = \frac{t_{P_1}^3}{3!}, \quad t_{0,P_1} \leq t \leq t_{1,P_1}, \quad (10)$$

$$f_{P_1}^{[3]}(t_{P_1}) = -\frac{t_{P_1}^3}{3!} + t_{P_1}^2 - t_{P_1} + \frac{1}{3}, \quad t_{1,P_1} \leq t_{P_1} \leq t_{2,P_1}.$$

The transitions from the coordinate systems of the element P_1 to the coordinate systems of the element P_2 do not preserve the form (10). For example, if under the transformation

$$t_{P_2} = t_{P_1} + 2, \quad f_{P_2}^{[3]} = -f_{P_1}^{[3]} + 1$$

the perspective is changed from the coordinate system of the element P_1 to a coordinate system of the element P_2 , then it turns out that the description of the dynamics (10) is not invariant

$$f_{P_1}^{[2]}(t) = f_{P_2}^{[3]}(t_{P_2}) = \frac{t_{P_2}^3}{3!} - t_{P_2}, \quad t_{1,P_2} \leq t_{P_1} \leq t_{2,P_2}$$

due to the additional linear term $-t_{P_2}$.

Therefore, on scale level 3 arithmetic determines the different dynamics of the elements P_1 and P_2 . Information about the difference might be obtained from two observers positioned at the coordinate system of the element P_1 and the coordinate system of the element P_2 respectively. As one observer would report about the dynamics of the element P_1 and the other about the dynamics of the element P_2 , we could find the difference and interpret it through the existence of a gauge force F acting on the element P_2 in its coordinate system to the effect of the linear term $\chi(F) = -t_{P_2}$

$$f_{P_2}^{[3]}(t_{P_2}) = \frac{t_{P_2}^3}{3!} - \chi(F),$$

$$t_{0,P_2} \leq t_{P_2} \leq t_{1,P_2}.$$

The introduction of the gauge force F restores the local symmetry

$$f_{P_2}^{[3]}(t_{P_2}) = \frac{t_{P_2}^3}{3!}, \quad t_{0,P_2} \leq t_{P_2} \leq t_{1,P_2} \quad (11)$$

as we can see comparing (10) and (11).

The results can be schematically expressed as follows

$$\text{Arithmetic} \rightarrow$$

$$\begin{aligned} & \text{Prime integer relations in control} \\ & \text{of correlation structures of complex systems} \leftrightarrow \\ & \text{Global symmetry: geometric patterns in control} \\ & \text{of the dynamics of complex systems} \rightarrow \\ & \rightarrow \text{Not locally invariant descriptions} \\ & \text{of embedded parts of complex systems} \leftrightarrow \\ & \leftrightarrow \text{Gauge forces to restore local symmetries} \end{aligned}$$

To discuss how the gauge forces resulting from the breaking of the global symmetry could be quantitatively classified, we consider whether the scale levels in our description could be subdivided into groups of successive scale levels so that the description of complex systems would be group scale invariant. If it would be the case then the classification of the gauge forces could be made possible by focusing on any group of scale levels. Every other group would have the same classification although expressed in its own terms.

Remarkably, as far as the self-organization process of prime integer relations is concerned, it turns out that the scale levels can be subdivided into groups of *three* successive levels, where the description of complex systems remains the same. In particular, using the renormalization group transformation applied to ε, δ

$$\varepsilon' = 2^3 \varepsilon, \quad \delta' = \varepsilon^3 \delta,$$

and elements P_1, P_2, \dots

$$P'_1 = ((P_1 \leftrightarrow P_2) \leftrightarrow (P_3 \leftrightarrow P_4))$$

$$\leftrightarrow ((P_5 \leftrightarrow P_6) \leftrightarrow (P_7 \leftrightarrow P_8)),$$

$$P'_2 = ((P_9 \leftrightarrow P_{10}) \leftrightarrow (P_{11} \leftrightarrow P_{12}))$$

$$\leftrightarrow ((P_{13} \leftrightarrow P_{14}) \leftrightarrow (P_{15} \leftrightarrow P_{16})),$$

$$\dots \dots \dots$$

we can obtain the same description of complex systems on scale levels 4, 5, 6 as for complex systems on scale levels 1, 2, 3. The difference is that the description on scale levels 4, 5, 6 is given in terms of ε' and δ' with parts P'_1, P'_2, \dots as the elements, while on scale levels 1, 2, 3 it is given in terms of ε, δ and elements P_1, P_2, \dots . The situation repeats for scale levels 7, 8, 9 and so on [3].

VI. CONCLUSIONS

In the paper we have presented results to develop an irreducible theory of complex systems in terms of self-organization processes of prime integer relations. Based on the integers and controlled by arithmetic only the self-organization processes can describe complex systems by

information not requiring further explanations. The following properties have been revealed.

First, the description points to a special type of correlations that do not depend on the distances between parts, local times and physical signals. Apart from the time aspect such correlations are known in quantum physics and attributed to quantum entanglement [5]. Thus, the description proposes a perspective on quantum entanglement suggesting to include the time aspect into the agenda.

Second, through a concept of structural complexity the description computationally reveals the possibility of a general optimality condition of complex systems. According to the optimality condition the optimal result can be obtained as long as the structural complexity of the system is properly related with the structural complexity of the problem. The experiments also indicate that the performance of a complex system may behave as a concave function of the structural complexity. Therefore, once the structural complexity would be controlled as a single entity, the optimization of a complex system could be potentially reduced to a one-dimensional concave optimization irrespective of the number of variables involved in its description.

In the search to identify a mathematical structure underlying optimal quantum algorithms the majorization principle emerges as a necessary condition for efficiency in quantum computational processes [9]. A connection between the majorization principle in quantum algorithms and the optimality condition has been identified. While the quantum majorization principle suggests that the computational process should stop when the probability distribution is maximally ordered, it does not however specify what this order actually means in the context of a particular problem. At the same time our approach is clear on this matter: to obtain the performance maximum the computational process should stop when its structural complexity is in a certain relationship with the structural complexity of the problem.

Third, the description introduces a global symmetry of complex systems that belongs to the system as a whole, but does not necessarily apply to its embedded parts. The breaking of the global symmetry may be interpreted through the existence of gauge forces. There is no further need to explain why the resulting gauge forces exist the way they do and not even slightly different as they are fully determined by arithmetic.

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